Sample IPC model

\[ M_4 = \langle W_4, I_4, <_4, U_4, V_4 \rangle, \text{ where} \]
\[ a. \ W_4 = \{w', w''\}, \ I_4 = \{i', i'', i'''\}, \]
\[ <_4 = \{\langle i', i''\rangle, \langle i'', i'''\rangle, \langle i', i'''\rangle\}, \ U_4 = \{a, b, c\} \]
\[ b. \ V_4(j) \text{ is a constant function such that for any } w \in W_4 \text{ and} \]
\[ i \in I_4, \ V_4(j)(\langle w, i \rangle) = a. \text{ Furthermore,} \]
\[ V_4(m) = w'' \begin{array}{ccc}
        b & b & a \\
      \hline \\
    w' & a & a & c \\
        i' & i'' & i''' \\
\end{array} \]
\[ V_4(P) = w'' \begin{array}{ccc}
        \{b, c\} & \{a, b\} & \{a\} \\
      \hline \\
    a, b, c & a, b & \{c\} \\
        i' & i'' & i''' \\
\end{array} \]

Evaluate \( \mathcal{P}\mathcal{P}(m) \) w/r t \( <w', i''> \)

Evaluate \( \mathcal{F}\mathcal{P}\mathcal{P}(m) \) w/r t \( <w', i''> \)

Evaluate \( \mathcal{F}\mathcal{P} \mathcal{P}(m) \) w/r t \( <w', i''> \)
INTENSIONALITY (2)
Another set theory convention

• Related to set intersection:

• Generalized intersection
  • Set that contains all elements that belong to every member in a set
  • Examples:
    \[ P = \{ \{a,b,c,d\}, \{a,b,d\}, \{a,d\} \} \]
    \[ \cap P = \{\{a,d\}\} \]

\[ P = \{ \{a,b\}, \{c,d\}\} \]
\[ \cap P = \emptyset \]
Other intensional phenomena

- Modality
- Complementation
  - Finite
  - Infinitival, gerundial
- Belief sentences
  - Problem: closed-world (or omniscience) assumption
Modality

- Making some kind of judgment on the truth value of a predicate

- Auxiliary verbs: can, should, must, would, might, could, etc.
  Fred runs 2 miles every day.
  Fred can run 2 miles every day.

- Adjectives: solvable, doable, mortal, fragile, forgetful, etc.

- Sentential adverbs: possibly, probably, for sure, etc.

- Adjectival predicates: is able, is possible, etc.
Semantics of modality

• Syntax: put modal aux verbs into the T node
• Operators so far (□, ◊) alone don’t quite give us the proper semantics.
  • Pavarotti can’t sing. 
    \( \neg \diamond \operatorname{sing}(P) \) isn’t quite right: he might just have a cold.
• Modals are context dependent (e.g. here, that Pavarotti can actually sing, just not if he has a cold)
  • They require information from the conversational background

• Epistemic modality: the judgment involves ability, possibility
• Deontic modality: the judgment involves permission
• Fred may not come to my party. (ambiguous)
The modal base

- Root/circumstantial use of modality: tied to the context
- Conversational background: relevant facts, what’s known, what’s polite, participants’ goals, agendas, etc.
- Semantics of modals specifies:
  - What kinds of backgrounds a modal admits
  - What modal relation is associated with it

- Each background: set of propositions; this set determines a set of worlds in which all propositions are true
  - This set of worlds is the modal base
g() to the rescue (again)

• We need to be able to map between circumstances and sets of propositions
• Endow g() with the power to do that
• Common ground at \(<w, i>\): \(p_1, p_2, p_3, \ldots p_n\)
• Then \(g(<w, i>) = \{p_1, p_2, p_3, \ldots p_n\}\)
• This set of propositions (i.e. \(\{p_1, p_2, p_3, \ldots p_n\}\)) determines the set of circumstances \(\{<w',i'>: \text{for every } p \in \{p_1, p_2, p_3, \ldots p_n\}, \ <w',i'> \in p\}\)
  • Call this the modal base at \(<w, i>\)
  • Defined as \(\cap \{p_1, p_2, p_3, \ldots p_n\}\), which is the same as \(\cap g(<w,i>)\)
  • \(\cap A\): set of objects that belong to all the sets in \(A\)
Truth value of modals

- For “must TP”: check whether the proposition from TP is entailed by the propositions in $g(<w,i>)$

- Possible worlds approach:
  1. Intersect all propositions in $g(<w,i>)$ with one another (i.e. $\cap$)
  2. Check whether the result from 1), which is a set of worlds, is a subset of the set of worlds in which TP is true.

- For “can TP”: check whether $\cap$ is nonempty.
Complementation (sentence embedding)

- “that” is a complementizer
- Just use CP, C’, C
- Embedded sentence: proposition

\[
\begin{align*}
\text{a. } & \text{CP} \rightarrow \text{C TP} \\
\text{b. } & \text{VP} \rightarrow \text{V CP}
\end{align*}
\]

a. Loren believes that Pavarotti is hungry.
b. 

```
TP
   /\  
  /  
NP T
  /  
T V
  /  
V CP
  /  
C TP
  /  
NP T
  /  
T VP
  /  
PRES believe that Pavarotti be hungry
```

Loren FRES believe that Pavarotti FRES be hungry
V for modality and the common ground

For any \( \langle w, i \rangle \), \( V(\text{believe}) (\langle w, i \rangle) = \{ \langle u, p \rangle : u \text{ is an individual and } p \text{ is a proposition, and } p \text{ is true in all those } w', i', \text{ compatible with what } u \text{ believes in } w \text{ at } i \} \)

- Denotation of verb: set of order pairs \( <u,p> \)
  - \( u \) is, as always, an individual from the domain of discourse
  - \( p \) is, as always, a proposition

- \( [[ \text{believe } ]]^{M,w,i,g} = \{ <\text{Mary, happy(John)>}, <\text{Fred, ugly(Fido)>}, <\text{Joe, tasty(x)} & \text{cheerios(x)>} \} \)
Semantics: intension of “believe”

a. $[[\text{CP that TP}]]_{M,w,i,g} = p$, where for any world $w'$ and any time $i'$, $p(<w', i'>) = 1$ iff $[\text{TP}]_{M,w',i',g} = 1$

b. $[[\text{vp believe CP}]]_{M,w,i,g} = \{u : <u, [\text{CP}]_{M,w,i,g}> \in [\text{believe}]_{M,w,i,g}\}$
Working it out

(a) \((64)\) \(M, w, i, g = 1 \iff \left(\left[\text{NP Loren}\right]^{\text{M, w, i, g}} \in \left[\text{VP believes that Pavarotti is hungry}\right]^{\text{M, w, i, g}}\right) \) By (39c)

(b) \(\left[\text{NP Loren}\right]^{\text{M, w, i, g}} \in \left[\text{VP believes that Pavarotti is hungry}\right]^{\text{M, w, i, g}} \iff \left[\text{NP Loren}\right]^{\text{M, w, i, g}} \in \{u : \left< u, \left[\text{CP that Pavarotti is hungry}\right]^{\text{M, w, i, g}} \in \left[\text{believe}\right]^{\text{M, w, i, g}} \right> \} \) By (67b)

(c) \(\left[\text{NP Loren}\right]^{\text{M, w, i, g}} \in \{u : \left< u, \left[\text{CP that Pavarotti is hungry}\right]^{\text{M, w, i, g}} \in \left[\text{believe}\right]^{\text{M, w, i, g}} \right> \} \iff \left< \left[\text{NP Loren}\right]^{\text{M, w, i, g}}, \left[\text{CP that Pavarotti is hungry}\right]^{\text{M, w, i, g}} \right> \in \left[\text{believe}\right]^{\text{M, w, i, g}} \) By definition of \(\in\)

(d) \(\left< \left[\text{NP Loren}\right]^{\text{M, w, i, g}}, \left[\text{TP that Pavarotti is hungry}\right]^{\text{M, w, i, g}} \right> \in \left[\text{believe}\right]^{\text{M, w, i, g}} \iff \left< \text{Loren, p} \right> \in V(\text{believe})(\left< w, i \right>), \text{ where for any } w', i', p(\left< w', i' \right>) = \left[\text{TP Pavarotti is hungry}\right]^{M, w', i', g} \) By (67a)

(e) \(\left< \text{Loren, p} \right> \in V(\text{believe})(\left< w, i \right>) \iff \text{ in all of Loren's belief worlds, Pavarotti is hungry} \) By (66)
Semantics of $^\land$

a. If $\psi$ is a well-formed formula, $^\land\psi$ is a propositional term.

b. $\langle [^\land\psi]_{M,w,i,g} = p, \text{ where for any } w', i', p(<w',i'>) = [\psi]_{M,w',i',g} \rangle$
Ambiguity

a. Bond believes that a student in that class is a spy.
b. \[[[\text{a student in that class}]_i \text{Bond believes that} \quad [\text{TP } e_i \text{ is a spy}]]\]
c. \[[\text{Bond believes that} \quad [\text{TP } [\text{a student in that class}]_i \quad [e_i \text{ is a spy}]]\]

- Two TP’s (because of the embedded clause), hence two adjunction sites possible
- Intensional operator: ^

\[
a. \exists x [S(x) \land \text{BELIEVE}(B, {}^\text{spy}(x))] \\
b. \text{BELIEVE}(B, {}^\exists x [S(x) \land \text{spy}(x)])
\]
**de dicto vs. de re**

- de dicto: “about what is said”
- de re: “about the thing”

(74)  
\[
\begin{align*}
\text{a. Some politician will address every rally in John’s district.} \\
\text{b. Some politician thinks that he will address every rally in John’s district.}
\end{align*}
\]

(75)  
\[
\begin{align*}
\text{a. } & \exists x[\text{politician}(x) \land \forall y[\text{rally}(y) \rightarrow \text{address}(x, y)]] \\
\text{b. } & \forall y[\text{rally}(y) \rightarrow \exists x[\text{politician}(x) \land \text{address}(x, y)]]
\end{align*}
\]

(76)  
\[
\forall y[\text{rally}(y) \rightarrow \exists x[\text{politician}(x) \land \text{think}(x, ^\text{address}(x, y))]]
\]

(77)  
\[
\begin{align*}
\text{a. } & \exists x[\text{politician}(x) \land \forall y[\text{rally}(y) \rightarrow \text{think}(x, ^\text{address}(x, y))]] \\
\text{b. } & \exists x[\text{politician}(x) \land \text{think}(x, ^\forall y[\text{rally}(y) \rightarrow \text{address}(x, y)])]
\end{align*}
\]
Untensed complements

• Subject control:
  • John tried to play tennis.
    John tried playing tennis.
• \([_{\text{TP}} \text{John}_n \text{ tried \left[_{\text{CP}} \emptyset \left[_{\text{TP}} \text{PRO}_n \text{ to play tennis}\right]\right]}\] \([_{\text{TP}} \text{John}_n \text{ tried \left[_{\text{CP}} \emptyset \left[_{\text{TP}} \text{PRO}_n \text{ playing tennis}\right]\right]}\])
• try(j, ^{\text{play_tennis}}(j))
Enriching the model for speakers

A model $M$ for $F_3$ is a sextuple of the form $\langle W, I, <, S, U, V \rangle$, where

a. $W, I, <, U$ are as before.

b. $S \subseteq U$ is a set of speakers.

c. $V$ is a function that assigns to each constant $\alpha$ an intension of the right type, where intensions are now functions from $W \times I \times S$; in particular, for any $w \in W, i \in I, \text{and } s \in S$,

$$V(I)(\langle w, i, s \rangle) = s.$$ 

a. If $\alpha$ is a basic constant, then $\sem{\alpha}^M_{w, i, s, g} = V(\alpha)(\langle w, i, s \rangle)$

b. If $\alpha$ is a trace or a pronoun, then $\sem{\alpha}^M_{w, i, s, g} = g(\alpha)$

c. $\sem{[[S \text{ NP Pred}]]}^M_{w, i, s, g} = 1$ iff $\sem{[\text{NP}]}^M_{w, i, s, g} \in \sem{[\text{Pred}]}^M_{w, i, s, g}$. 
Spurious ambiguity

• Some possible construals never come to mind, or don’t differ noticeably from other ones.

• Not all glasses were made in Italy.
  • QNP, neg, past tense, so 3! = 6 possible orderings at LF
  • Some are spurious
Lambda conversion

- An operation that permits variable substitution into open predicates
- \( \neg \text{married}(\text{John}) \land \text{male}(\text{John}) \land \text{adult}(\text{John}) \)
- \( (\lambda x. (\neg \text{married}(x) \land \text{male}(x) \land \text{adult}(x)))(\text{John}) \)
- In an expression \( \lambda x. Y(z) \), replace all occurrences of the variable \( x \) in the expression \( Y \) with \( z \).
- \( (\lambda x. \text{hungry}(x))(\text{John}) \rightarrow \text{hungry}(\text{John}) \)
Lexical items and predication

• …sneezed $\to \lambda x. (\mathcal{P} \text{sneeze}(x))$
• …saw… $\to \lambda y. \lambda x. (\mathcal{P} \text{see}(x,y))$
• John sneezed.
  John, $\lambda x. (\mathcal{P} \text{sneeze}(x))$
  $\lambda x. (\mathcal{P} \text{sneeze}(x))$ (John)
  $\mathcal{P}$ sneeze(John)
More lambda formulas

• ... laughed and is not a woman
  \( \lambda x. (\text{laughed}(x) \land \neg \text{woman}(x)) \)
• ... respects himself
  \( \lambda x. \text{respect}(x,x) \)
• ...respects and is respected by...
  \( \lambda y. \lambda x. [\text{respect}(x,y) \land \text{respect}(y,x)] \)