IMPERATIVES
Background

• Used to issue commands; often associated with mood in syntax
• English: subject is not specified, though vocatives are possible
• Some imperative-like constructions don’t give orders:
  • Hortatives, optatives, answers to questions, etc.
• When they do, various functions:
  • Command, suggestion, advice, invitation, request, granting permission, instruction, etc.
• How much of this is semantic? How much should we relegate to pragmatics?
Semantic characterization

• Don’t seem to have a truth value in the traditional sense
• Other relations do seem to hold, though, such as:

  • Entailment
    
    “close the window!” “shut the door!”
    Therefore: “close the window and shut the door!”

  • Inference
    
    “Heal the sick!” “John is sick”
    Therefore: “Heal John”

• We can use propositional logic to characterize such relationships
Examples

• Propositional logic connectives: $\neg$, $\&, V, \rightarrow$

(7)  a. “Close the window and shut the door!”
    b. “Watch television, or go to the beach!”
    c. “Don’t watch television!”
    d. “If you have finished your homework, do the washing up!”
    e. “Have another drink, or you will be thirsty!”
    f. “Have another drink and you will be happy!”
    g. “Have another drink and you will die!”

(8)  “Do not eat the cheese!”

(9)  a. “Turn on the light and close the curtains!”
    b. “Jump out of the window and land on the mattress!”

(10) a. “Turn on the light!” “Close the curtains!”
    b. “Jump out of the window!” “Land on the mattress!”

(12) “Go to the beach, or play in the park!”
(13) “Have some apple or bananas!”
(14) “Sleep on the bed, or on the couch!”
(21) “If you see John, say hello!”
(23) a. “Take another step and I will kill you.”
    b. “Take another step or I will kill you.”
    c. “Have more fruit or you will become ill.”
    d. “Have more fruit and you will become ill.”
    e. “Have more fruit and you will get better.”
(15) “Eat an apple.”
(16) “Eat apple A or eat apple B or eat apple C or…”
(17) a. “What do I need to do?”
    b. “Buy some teak or mahogany, depending on which is in stock.”
(18) a. “Which way should I go?”
    b. “Go north over the mountains or south along the coast [it depends on the time of year].”
Considerations

• Imperatives might have some connection to deontic modals…
  • … but don’t get too invested in this line of thought.
• Jørgensen’s Dilemma: what happens if imperatives support inference while lacking truth values
  
  \[
  \begin{align*}
  \text{(28)} & \quad \text{“Love your neighbour as you love yourself.” } \text{“Love yourself.”} \\
  & \quad \text{Therefore: “Love your neighbour.”} \\
  \text{(29)} & \quad \text{“Keep your promises.” } \text{“This is one of your promises.”} \\
  & \quad \text{Therefore: “Keep this promise.”}
  \end{align*}
  \]

• Ross’s Paradox: some imperatives entail others; but where do we draw the line?
  
  \[
  \begin{align*}
  \text{(30)} & \quad a. \quad \text{“Post the letter!”} \\
  & \quad b. \quad \text{“Post the letter or burn the letter!”}
  \end{align*}
  \]

• The Good Samaritan Paradox: imperatives carry implicit value judgments
  
  \[
  \begin{align*}
  \text{(32)} & \quad \text{“Help an injured man!”} \quad \text{(but obeying this order doesn’t entail injuring someone so you can help him!)}
  \end{align*}
  \]
Issues

• Semantics or pragmatics? Should be able to relate the two somehow…
• Entailment is at the core of several accounts.
  • Validity: entailment relations about what has been commanded
  • Satisfaction: entailment relations about the satisfaction conditions of commands
• Ontology: Can we use states of affairs (i.e. circumstances) to check whether an imperative has been satisfied? How does agency fit into imperatives?
• Frameworks: to-do lists of discourse obligations? Possible worlds?
Approaches

• Lewis: modality, possible worlds, master/slave relationship between accessible worlds
  • Doesn’t overtly model combination of imperatives with propositions
• Kratzer: modal subordination: some context sensitivity, ranking possible worlds (modal base + ordering source) wrt relevance, desirability, ethical and legal obligations, etc.
  • Doesn’t capture appropriate behavior in all cases
• Programming languages analogy: pre-conditions, actions, post-conditions
  • Negation is problematic: what is not-biting-the-apple?
• Dynamic-pragmatic approaches: discourse obligation: agent has to accept/reject some imperative to comply; goals, plans, ethics, etc. come into play
• Ontological reduction problem: “Everything is just a set.”
Fox’s proposal

• Focus on how inference rules are actually rules concerning judgments
  • Can use propositional logic:

(42) \[ \frac{a \quad b}{a \land b} \]

(43) \[ \frac{a \text{ True} \quad b \text{ True}}{(a \land b) \text{ True}} \]

(44) \[ \frac{a \text{ Prop} \quad b \text{ Prop} \quad (a \land b) \text{ Prop} \quad a \text{ True} \quad b \text{ True}}{(a \land b) \text{ True}} \]

(45) a. \[ \frac{a \text{ Prop} \quad b \text{ Prop}}{(a \land b) \text{ Prop}} \]

b. \[ \frac{a \text{ Prop} \quad b \text{ Prop} \quad a \text{ True} \quad b \text{ True}}{(a \land b) \text{ True}} \]

• Judgments: truth value, plus: what’s been commanded, what’s (not) been satisfied, what’s inconsistent, whether an agent is acting incoherently
Sequent calculus

• Deriving entailment (⊢) from a given context (Γ)

(46) a. \[ \Gamma \vdash a \text{ Prop} \quad \Gamma \vdash b \text{ Prop} \]
\[ \Gamma \vdash (a \land b) \text{ Prop} \]

b. \[ \Gamma \vdash a \text{ Prop} \quad \Gamma \vdash b \text{ Prop} \quad \Gamma \vdash \text{ a True} \quad \Gamma \vdash \text{ b True} \]
\[ \Gamma \vdash (a \land b) \text{ True} \]

(47) a. \[ \Gamma \vdash a \text{ Prop} \quad \Gamma \vdash b \text{ Prop} \]
\[ \Gamma \vdash (a \rightarrow b) \text{ Prop} \]

b. \[ \Gamma \vdash a \text{ Prop} \quad \Gamma \vdash b \text{ Prop} \quad \Gamma, a \text{ True} \vdash \text{ b True} \]
\[ \Gamma \vdash (a \rightarrow b) \text{ True} \]

c. \[ \Gamma \vdash a \text{ Prop} \quad \Gamma \vdash b \text{ Prop} \quad \Gamma \vdash a \rightarrow b \text{ True} \quad \Gamma \vdash a \text{ True} \]
\[ \Gamma \vdash \text{ b True} \]

(48) a. \[ \Gamma \vdash a \text{ Prop} \quad \Gamma \vdash b \text{ Prop} \]
\[ a \lor b \text{ Prop} \]

b. \[ \Gamma \vdash a \text{ True} \quad \Gamma \vdash b \text{ Prop} \]
\[ \Gamma \vdash a \lor b \text{ True} \]

c. \[ \Gamma \vdash a \text{ Prop} \quad \Gamma \vdash b \text{ True} \]
\[ \Gamma \vdash a \lor b \text{ True} \]

d. \[ \Gamma, a \text{ True} \vdash c \text{ True} \quad \Gamma, b \text{ True} \vdash c \text{ True} \quad \Gamma \vdash a \lor b \text{ True} \]
\[ \Gamma \vdash c \text{ True} \]
The framework: judgments

- \( p \) a proposition, \( i \) an imperative, \( \sigma \) a subject, \( \bot \) falsum/inconsistency
- Don’t bother indicating explicity \( \Gamma \): just assume it

<table>
<thead>
<tr>
<th></th>
<th>Propositions</th>
<th>Imperatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Syntax”</td>
<td>( p ) Prop</td>
<td>( i ) Imp</td>
</tr>
<tr>
<td>“Semantics”</td>
<td>( p ) True</td>
<td>( i ) Satisfied(_\sigma)</td>
</tr>
<tr>
<td></td>
<td>( p ) False</td>
<td>( i ) unSatisfied(_\sigma)</td>
</tr>
</tbody>
</table>

- Satisfaction: a judgement about whether an imperative has been satisfied

\[
\text{Satisfied}_\sigma \quad \text{unSatisfied}_\sigma
\]

\[
\bot
\]
Conjunction (Sat)

\[
\begin{align*}
\frac{a \text{ Imp} \quad b \text{ Imp}}{(a \land b) \text{ Imp}} \\
\frac{a \text{ Satisfied}_\sigma \quad b \text{ Satisfied}_\sigma}{(a \land b) \text{ Satisfied}_\sigma} \quad & \quad \text{a.} \\
\frac{a \text{ unSatisfied}_\sigma}{(a \land b) \text{ unSatisfied}_\sigma} \quad & \quad \text{b.} \\
\frac{b \text{ unSatisfied}_\sigma}{(a \land b) \text{ unSatisfied}_\sigma} \quad & \quad \text{c.} \\
\frac{(a \land b) \text{ Satisfied}_\sigma}{(a \land b) \text{ Satisfied}_\sigma} \quad & \quad \text{d.} \\
\frac{a \text{ Satisfied}_\sigma}{(a \land b) \text{ Satisfied}_\sigma} \quad & \quad \text{e.} \\
\frac{(a \land b) \text{ unSatisfied}_\sigma}{a \text{ Satisfied}_\sigma} \quad & \quad \text{f.} \\
\frac{b \text{ unSatisfied}_\sigma}{a \text{ Satisfied}_\sigma} \quad & \quad \text{g.} \\
\frac{b \text{ Satisfied}_\sigma}{(a \land b) \text{ unSatisfied}_\sigma} \quad & \quad \text{h.} \\
\frac{a \text{ unSatisfied}_\sigma}{(a \land b) \text{ unSatisfied}_\sigma} \\
\end{align*}
\]
Basic free choice (Sat)

\[
\begin{align*}
(a \Imp b) \Imp (a \lorFC b) \Imp & \\
\quad a \text{ Satisfied}_\sigma \quad b \text{ unSatisfied}_\sigma & \quad (a \lorFC b) \text{ Satisfied}_\sigma \\
\quad b \text{ unSatisfied}_\sigma & \quad a \text{ Satisfied}_\sigma \quad b \text{ Satisfied}_\sigma \\
\quad (a \lorFC b) \text{ Commanded}_\alpha & \\
\quad a \text{ unSatisfied}_\sigma \\
\quad b \text{ unSatisfied}_\sigma & \\
\quad (a \lorFC b) \text{ unSatisfied}_\sigma \\
\quad a \text{ Satisfied}_\sigma \\
\quad b \text{ unSatisfied}_\sigma & \\
\quad (a \lorFC b) \text{ Satisfied}_\sigma \\
\quad a \text{ unSatisfied}_\sigma \\
\quad b \text{ Satisfied}_\sigma & \\
\quad (a \lorFC b) \text{ unSatisfied}_\sigma \\
\quad a \text{ unSatisfied}_\sigma & \\
\quad b \text{ unSatisfied}_\sigma
\end{align*}
\]
Negation (Sat)

\[
\begin{align*}
\frac{a \text{ Imp}}{\neg a \text{ Imp}} \\
\frac{a \text{ Satisfied}_\sigma}{\neg a \text{ unSatisfied}_\sigma} \\
\frac{\neg a \text{ Satisfied}_\sigma}{a \text{ unSatisfied}_\sigma} \\
\frac{a \text{ unSatisfied}_\sigma}{\neg a \text{ Satisfied}_\sigma}
\end{align*}
\]
Conditionals (Sat)

\[
\begin{align*}
\text{Prop} & \quad \text{Imp} \\
\iff (p \rightarrow a) & \quad \text{Imp} \\
\text{a. } p \quad \text{True} & \quad a \quad \text{Satisfied}_\sigma \quad \Rightarrow \quad (p \rightarrow a) \quad \text{Satisfied}_\sigma \\
\text{c. } p \quad \text{True} & \quad (p \rightarrow a) \quad \text{Satisfied}_\sigma \quad \Rightarrow \quad a \quad \text{Satisfied}_\sigma \\
\text{b. } p \quad \text{True} & \quad a \quad \text{unSatisfied}_\sigma \quad \Rightarrow \quad (p \rightarrow a) \quad \text{unSatisfied}_\sigma \\
\text{d. } p \quad \text{True} & \quad (p \rightarrow a) \quad \text{unSatisfied}_\sigma \quad \Rightarrow \quad a \quad \text{unSatisfied}_\sigma
\end{align*}
\]
Pseudo-Or (Sat)

\[
\begin{align*}
Pseudo-Or & \\
a \quad \text{Imp} \quad p \quad \text{Prop} & \\
      & \quad \text{(a \lor p) Imp} \\
a \quad \text{Satisfied}_\sigma & \\
(a \lor p) \quad \text{Satisfied}_\sigma & \\
(a \lor p) \quad \text{Satisfied}_\sigma & \\
      & \quad \text{(a \lor p) unSatisfied}_\sigma \\
a \quad \text{Satisfied}_\sigma & \\
(a \lor p) \quad \text{unSatisfied}_\sigma & \\
      & \quad \text{(a \lor p) unSatisfied}_\sigma
\end{align*}
\]
Truth

• Conclusions about truth value
• Pseudo-imperatives: similar to satisfaction, except that now we evaluate whether they’re true/false rather than evaluating whether they’ve been asserted (or commanded)
Pseudo-And (Truth)

- *Eat that and you’ll be sick.*

\[
Pseudo-And \\
\begin{array}{ccc}
\text{a Imp } & \text{p Prop} \\
\hline
\text{a } \land \text{ p Prop} \\
\end{array}
\]

\[
a \text{ Satisfied}_\sigma \, \text{p True} \\
\hline
\text{p True} \\
\]

\[
a \text{ Satisfied}_\sigma \, \text{p True} \\
\hline
\text{(a } \land \text{ p) True} \\
\]

\[
a \text{ Satisfied}_\sigma \, \text{p False} \\
\hline
\text{(a } \land \text{ p) False} \\
\]
Pseudo-Or (Truth)

\[
\begin{align*}
& a \text{ Imp } p \text{ Prop} \\
& \quad \quad a \lor p \text{ Prop} \\
& \quad \quad \quad (a \lor p) \text{ True } \quad a \text{ unSatisfied}_\sigma \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad p \text{ True} \\
& \quad a. \quad (a \lor p) \text{ True } \quad a \text{ unSatisfied}_\sigma \\
& \quad \quad \quad p \text{ True} \\
& \quad b. \quad (a \lor p) \text{ True } \\
& \quad c. \quad (a \lor p) \text{ True } \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{ Satisfied}_\sigma
\end{align*}
\]
Sequential commands

\[ a \text{ Imp} \quad b \text{ Imp} \]
\[ a \& T b \text{ Imp} \]

a. Initial Coherence
\[ (a \& T b) \text{ Commanded}_\alpha \quad \neg a \text{ Commanded}_\alpha \quad \alpha \text{ Incoherent} \]

b. Consequent Coherence (Strong)
\[ (a \& T b) \text{ Commanded}_\alpha \quad \neg b \text{ Commanded}_\alpha \quad \alpha \text{ Incoherent} \]

c. Consequent Coherence (Weak)
\[ a \text{ Satisfied}_\sigma \quad (a \& T b) \text{ Commanded}_\alpha \quad \neg b \text{ Commanded}_\alpha \quad \alpha \text{ Incoherent} \]

d. Satisfaction
i. \[ (a \& T b) \text{ Satisfied}_\sigma \]
   \[ a \text{ Satisfied}_\sigma \]
   ii. \[ (a \& T b) \text{ Satisfied}_\sigma \]
   \[ b \text{ Satisfied}_\sigma \]
   iii. \[ a \text{ Satisfied}_\sigma \text{ AND THEN } b \text{ Satisfied}_\sigma \]
   \[ (a \& T b) \text{ Satisfied}_\sigma \]

note the agent's incoherence
Obedience

- Transgression: when an authority $\alpha$ has commanded something that the subject $\sigma$ has failed to satisfy:

\[
\text{(45) Transgression}\quad \frac{\alpha \text{ Commanded}_\alpha \quad \alpha \text{ unSatisfied}_\sigma}{\text{I}_{\sigma,\alpha,\alpha}}
\]